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MODELLING COMBAT AS A SERIES OF MINIBATTLES

BY

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MODELLING COMBAT AS A SERIES OF MINIBATTLES

1. This is the fourth interim report on the study investigating the feasibility of modelling battalion level combat as a series of minibattles, sponsored by the US Army Research, Development and Standardisation Group (UK) under contract number DAJA45-86-C-0053. The work is also sponsored by the Directorate of Science (Land) (D.Sc(L)) of the UK Ministry of Defence under contract number D/ER1/9/4/2004/02/DSc(L).

OBJECTIVE OF THE STUDY

2. The current interest in network battle modelling arose from the analysis of the trial ' CHINESE EYE III ', [A,B], carried out by David Rowland and others at the UK Defence Operational Analysis Establishment (DOAE). The objective of the current investigation is to assess the utility of the networking concept as the basis for a model of battalion level combat. Such a model could be designed to be fast running and easy to set up - like many current highly aggregated Lanchester based models - and at the same time provide a more detailed and accurate representation of combat than is currently possible in Lanchester based models.

3. The original study proposal envisaged that the structure of the programme of work would be as follows:

a. the collection and analysis of data,

b. derivation of an appropriate methodology for generating networks,

c. the investigation of attrition methodologies,

d. development and verification of a combat model,

e. validation and assessment of the model.

This report will discuss progress to date in each of these aspects,and outline the proposed programme of future work.

DATA COLLECTION AND ANALYSIS

4. The objectives of this part of the study were twofold :

a. To establish the relationship - if any - between network structure and the terrain,mix of forces and tactics employed.

b. To assess the sensitivity of network structure to changes in the rules used to derive the network.

5. Results of this analysis are discussed at Annex A.The main conclusions to be drawn from this analysis are

a. The relation between scenario parameters and network structure is most significant for force ratio PDF's,and derivation of the force ratio PDF from relatively crude scenario data is clearly possible.

b. The relationship between scenario data and other network parameters is less pronounced and this suggests that quite detailed scenario data will be required in order to generate a representative network in a combat model.

6. The analysis of data from the ARCOMS trial is nearing completion, and a discussion of the results of this analysis will appear as a separate report.

METHODOLOGY FOR GENERATING A NETWORK.

7. Details of work in this area are at Annex B.

8. The current objective of the research team is to develop two prototype network combat models using

a. A force ratio PDF based approach

and

b. A method based on a family of inter-kill time PDF's

by the end of September 1988.Further network generation methodologies may be incorporated after the prototype model has been assessed.

ATTRITION METHODOLOGY

9. In a network based combat model,the forces will fight a number of small engagements and it is therefore important to use an attrition methodology appropriate to small force-on-force engagements.

10. Current work in this area is discussed at Annex C.

DEVELOPMENT OF A COMBAT MODEL

11. Work has now begun on the development of a prototype network combat model.A method for the resolution of networked combat already exists,in the form of the preset network model discussed in earlier reports,and initially this will form the basis of the prototype model,although the ultimate intention is to construct a model which can employ a variety of network generation and combat resolution methodologies.

FUTURE WORK

12. Future work will concentrate on the development of two prototype network combat models,and this development process should be completed by September 1988.

CONCLUSIONS

13. Progress is being made in each component of the work programme and the development of a combat model has now begun.

REFERENCES

A. Rowland D.,' The Local Anti-Armour Density Necessary to Defeat a WP Attack ',classified UK CONFIDENTIAL,DOAE Note Number 800/200 April 1981.

B. Rowland D.,' Field Trials and Modelling ',paper presented to the International Symposium on Advances in Combat Modelling,held at RMCS,Shrivenham,Swindon,UK, 4 - 7 September 1984.

ANALYSIS OF DATA : EXERCISE CHINESE EYE III

A1 The trial Chinese Eye III took place in Germany in the 1970's and consisted of a number of armour/anti-armour battles at Red Battalion,Blue Combat Team level.The trials involved tanks and guided weapons only.

A2 For each battle,each round fired was recorded,together with the firer's callsign and position,the target's callsign and position,the time of the event and the outcome of the engagement.

A3 The objective in analysing this data was to determine how the decomposition of a large battle into a series of smaller engagements is determined by the detail of a given scenario (ie. terrain,deployment,objectives) and to assess the extent to which this decomposition could be modelled statistically.To this end,a number of FORTRAN programs were constructed to produce statistics relating to battle structure and decomposition from the Chinese Eye data.The resulting output has been analysed using a PC-based statistical package and a discussion of the results of this analysis follows.

DATA ANALYSIS

A4 The output from the analysis programs consisted of the following for each minibattle :

1. node number
2. start time
3. end time
4. duration
5. total number of shots fired in minibattle
6. average range
7. initial number of blue weapons
8. initial number of red weapons
9. final number of blue weapons
10. final number of red weapons.
11. callsign of each weapon involved
12. last minibattle this weapon took part in

and, for each weapon, the sequence of minibattles in which that weapon was involved.

Force Ratio Data

A5 The force ratio statistics are summarised below, for each battle. Ten separate battle scenarios were suudied and are identified by a separate unique battle number.

TABLE 1. FORCE RATIO STATISTICS BY BATTLE NUMBER

battle	nodes	mean	mode	median	variance	st. dev.
4	25.0000	1.455238	1.000000	1.000000	1.382432	1.175769
5	16.0000	2.312500	2.000000	1.000000	2.329167	1.526161
6	23.0000	1.043478	1.000000	1.000000	0.260019	0.509920
7	17.0000	1.919608	1.800000	1.000000	1.167508	1.080513
8	19.0000	1.644737	1.500000	1.000000	0.661915	0.813582
12	20.0000	2.579762	2.285714	1.000000	3.791221	1.947106
13	8.0000	2.625000	1.500000	1.000000	4.267857	2.065879
14	14.0000	2.142857	2.000000	2.000000	1.362637	1.167321
18	29.0000	1.673563	1.000000	1.000000	1.653522	1.285894
19	39.0000	1.844017	1.000000	1.000000	2.256645	1.502213

The empirical cumulative distributions functions for force ratios in each battle are plotted in Figure 1.

A6 The above data and the distributions of force ratio suggest that different scenarios do indeed result in different distributions of force ratios.This assertion is supported by the following :

TABLE 2 : KOLMOGOROV - SMIRNOV 2-SAMPLE TEST : FORCE RATIO PDF'S

SIGNIFICANCE LEVEL = 5%

NULL HYPOTHESIS :

Force ratios in battles n,m are identically distributed.

sample	25	15	23	17	19	20	8	14	29	39
battle	4	5	6	7	8	12	13	14	18	19
4		R	R	R	R	R	R	R	R	R
5			R	R	R	R	A	R	R	R
6				R	R	R	R	R	R	R
7					R	R	A	R	R	R
8						R	A	R	R	R
12							A	R	R	R
13								R	R	R
14									R	R
18										R

R = Null hypothesis rejected
A = Null hypothesis not rejected.

In other words,with the exception of battle scenario 13 - in which the small sample size must cast doubt on any conclusions - none of the scenarios can be assumed to have identically distributed force ratios.

A7 In fact,with the exception of scenario 13,the force ratio pdf's are well described by Lognormal distributions.

A8 Rowland [A],in his original paper on the analysis of the Chinese Eye data, relates the mean local odds to the density ratio of red and blue forces.It is also possible to relate the mean engaged force ratio in a minibattle (EFR) to the density of blue forces - defining this to be the average separation of blue weapons systems,calculated using the Euclidean metric.Figure 2 shows this relationship.

A9 Rowland pointed out that the relation between density of forces and mean local odds was strongly influenced by phenomena which he described as lateral division of defence (LDD) and longitudinal division of attack (LDA).LDD occurs when the attacking red thrust is concentrated at a particular point - usually on a flank - and the blue defending force is divided by an obstacle or terrain feature.This results in a portion of the blue defenders being unable to engage the attacking units and hence in an increase in local odds.LDA occurs in scenarios where the attacking force is advancing across a series of transverse ridges when engaged by the defenders.This results in individual red weapons,or at most red platoons,being engaged by the defending force.The effect of this phenomenon on local odds will also be a function of red force density.The effect in the two scenarios considered is obscured by the fact that the red force density is similar for both scenarios.

A10 This influence is also apparent for the relation between density of blue forces and EFR, the upper dotted line in Figure 2 representing LDD and the lower, LDA. Comparison of the variance of the force ratio in a minibattle with the blue force density reveals a similar relation, although variances show a less consistent dependency on LDD and LDA.

A11 It is also possible to predict the expected force ratio, using a multiple linear regression procedure with the expected number of shots fired per weapon for blue and red as the independent variables. In addition, the variance of force ratio can be predicted - although less accurately - in terms of the variance of the number of shots fired per weapon.

Expected Minibattle Duration.

A12 Some variation in the expected duration of minibattles from scenario to scenario was noted, and this data is plotted against blue force density in Figure 3.

A13 The particularly long expected minibattle durations for scenarios 7 and 8 are associated with battles fought over open, gently rolling countryside. Scenarios 13 and 14 took place in relatively poor visibility, and therefore the length and duration of LOS is not a function of terrain only. Scenario 6 is an example of a reverse slope defensive deployment, which seems to account for the short average minibattle duration.

A14 No significant trend in minibattle duration as a function of battle time was apparent.

A15 Minibattle durations appear to be well described by negative exponential distributions.

Minibattle Initiation Times.

A16 The pdf's of minibattle start times are shown in Figure 4.

A17 Although there is a clear variation in the shape of the start time pdf from scenario to scenario, there seems to be little relation between this and identifiable features of the scenarios themselves. Most of the pdf's are distinctly bi- or tri-modal and show distinct phases where no minibattles are initiated at all. The absence of an obvious relation between start time and scenario characteristics is explained by the fact that the initiation of a minibattle depends on decisions taken by commanders of individual weapon systems, and this is a function of a number of random factors, in addition to terrain and the tactical situation.

Network Parameters.

A18 The main network parameters are the number of distinct minibattles or nodes, in the network and the number of links between nodes.

A19 The number of nodes is largely a function of the rules used to derive the network from the raw data,and not surprisingly,the number of nodes generated proves to be sensitive to certain variations in these rules. The selection of rules for generating a network is a subjective process,and appropriate rules can only be derived by analysis of the networks generated by a variety of different assumptions.As a result of analysis,some modifications have been made to the network generation rules employed,to allow more representative decompositions to be produced.

A20 The number of links between nodes in a network depends on both the scenario characteristics and on the network generation assumptions.Again,no pattern that relates in an obvious way to the scenario being analysed emerges,although the number of links per node appears to follow a binomial distribution for each scenario studied.

Conclusions

A21 The relation between force ratios and blue force density and the expected number of shots fired by a weapon suggests that pdf of force ratio can be specified for a given scenario by the use of a small number of relatively crude parameters.The same is true for minibattle durations.The remainder of the network parameters,however,show little simple dependency on such parameters.The implications of this for the development of a combat model are twofold :

a. If only a crude tactical/terrain description can be given,it will only be possible to specify the pdf's of force ratio and of minibattle duration with any certainty.Therefore,any combat model using such a terrain description will have to employ the network generation methodology based on force ratio pdf's discussed in Annex B.

b. Any of the alternative network generation methodologies described in Annexe B. will require a fairly detailed scenario description in order to function.This implies that an efficient method for scenario generation and editing will have to be developed,and a data base of scenarios,and perhaps also of derived networks,will have to be established.

Further Work

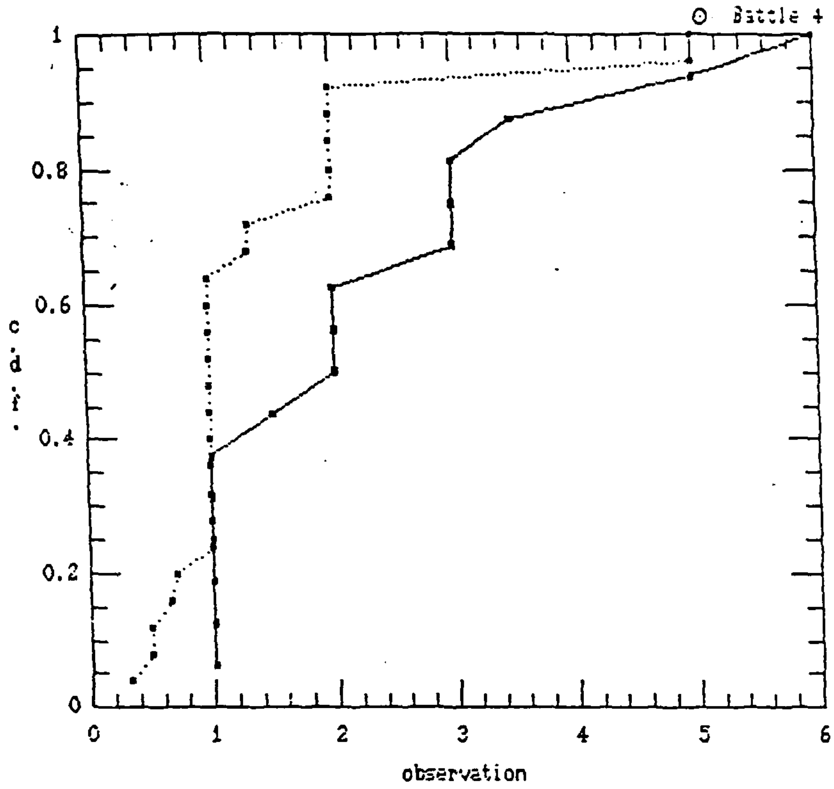
A22 A discussion of the results of analysis of data from the ARCOMS trial will appear as a separate report.At this stage,it seems likely that this will confirm the results of the Chinese Eye analysis.

A23 Further investigation of the relationship between battle structure and scenario type could be achieved by analysis of data from a computerised wargame such as JANUS-T.This has two advantages over the use of trials data.Firstly,a number of replications of the same battle could be played,and this would allow a rather better picture of the statistical properties of battle group level combat to be built up.Secondly,the effects of commander decision making could be more readily analysed.It is not feasible to undertake such a analysis as part of the current study,but a thourough statistical study of the results of battle group level combat could provide some useful results.

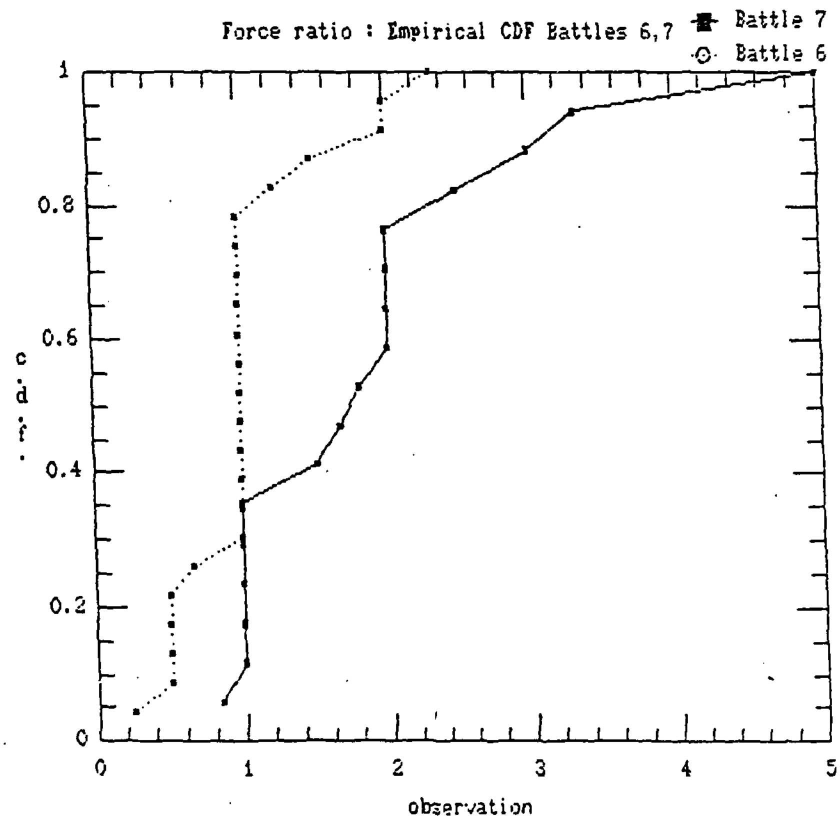
References

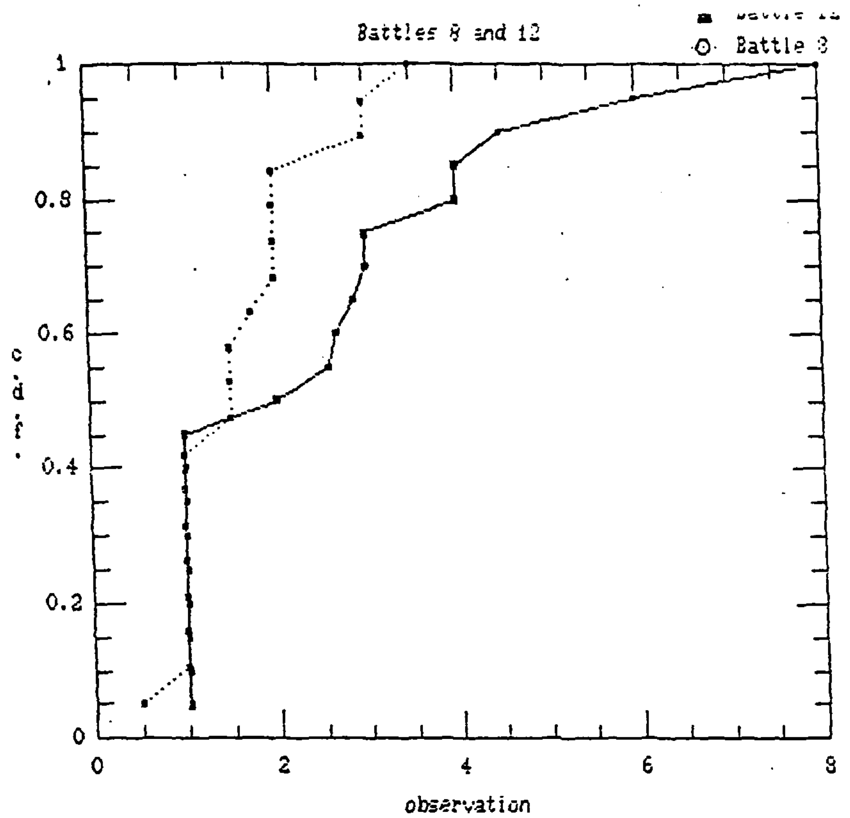
A. Rowland D.,' Field Trials and Modelling ',paper presented to the International Symposium on Advances in Combat Modelling,held at RMCS,Shrivenham,Swindon,UK, 4 - 7 September 1984.

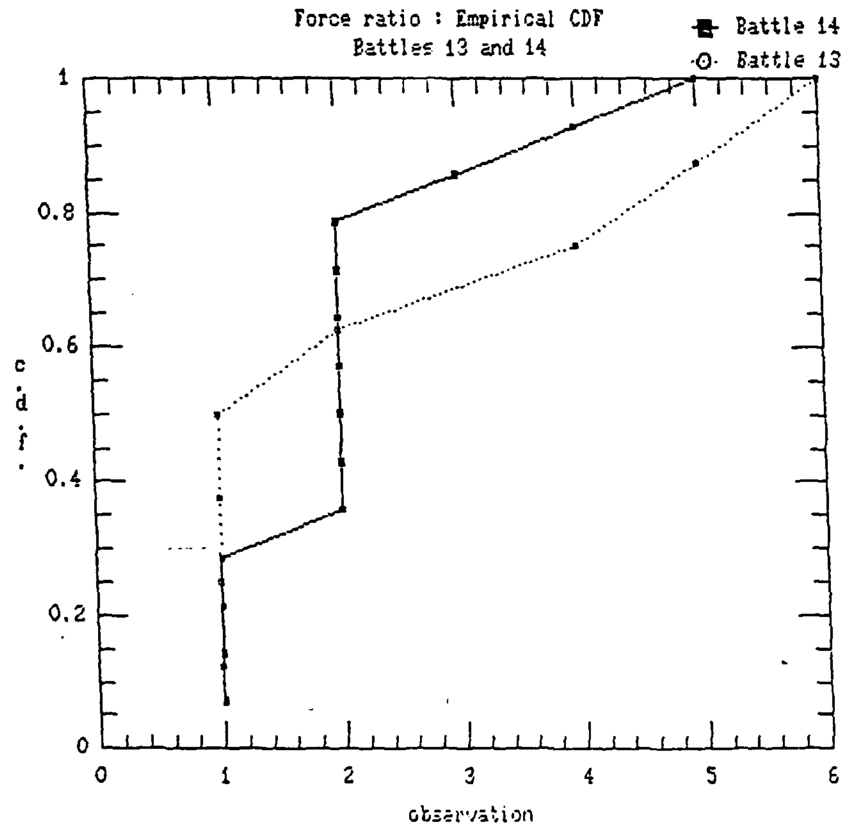
FIGURE 1.

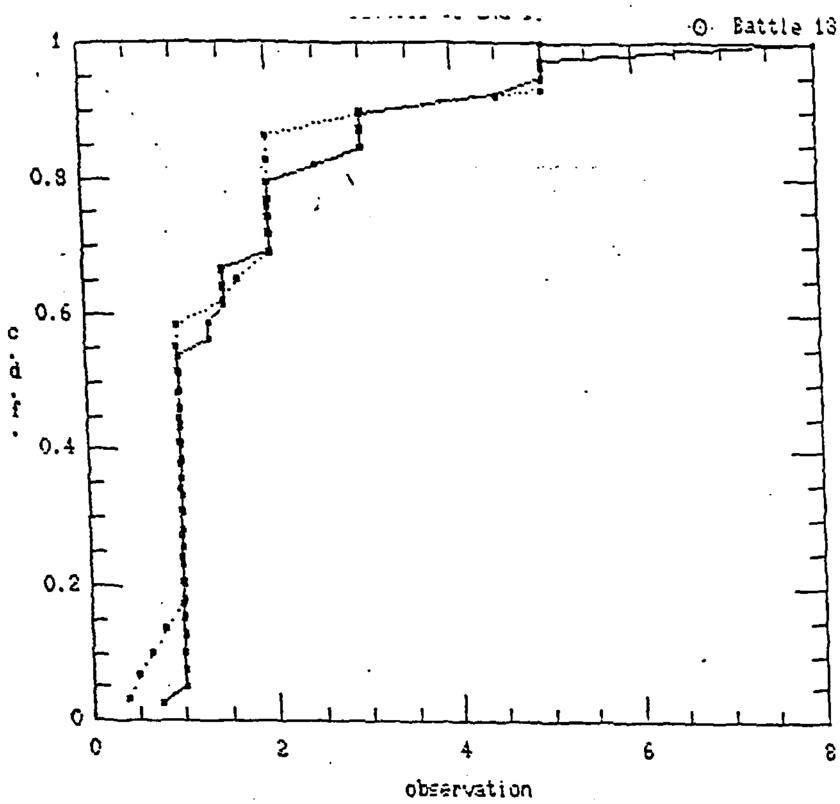


Force ratio : Empirical CDF Battles 6,7

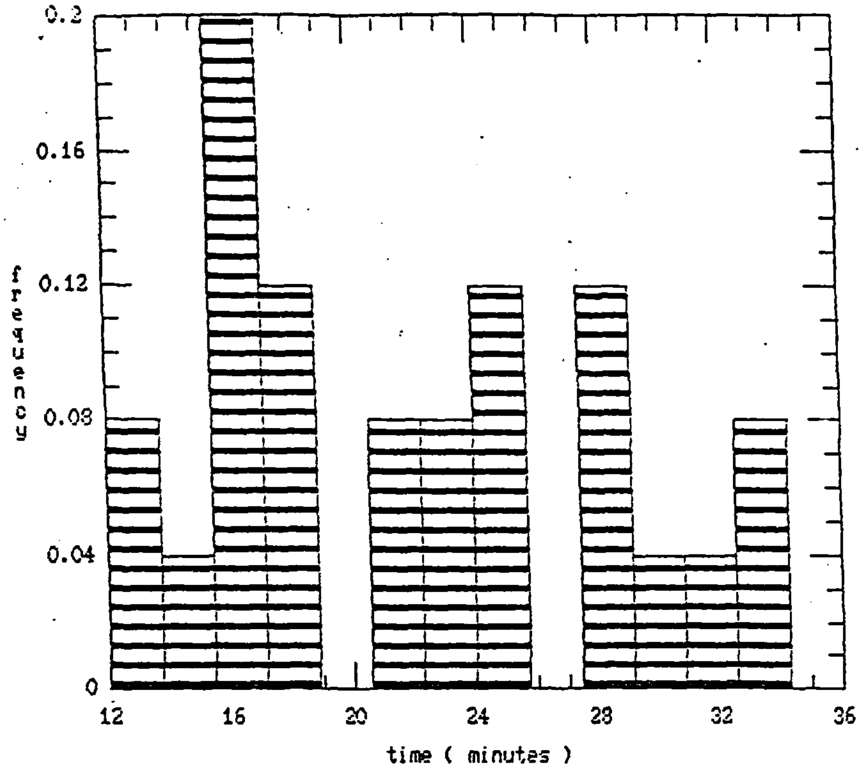



Battles 8 and 12
Battle 8
c.d.f.
observation
0
0.2
0.4
0.6
0.8
1
0
2
4
6
8


Force ratio : Empirical CDF
Battles 13 and 14
Battle 14
Battle 13
c.d.f.
observation
0
0.2
0.4
0.6
0.8
1
0
1
2
3
4
5
6

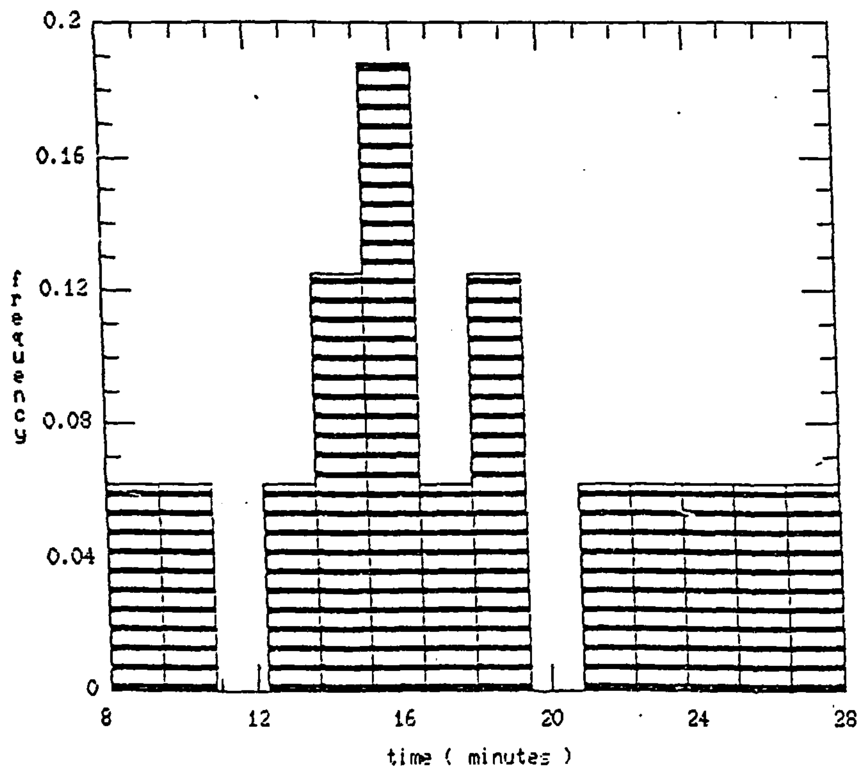


Battle 13
c.d.f.
observation

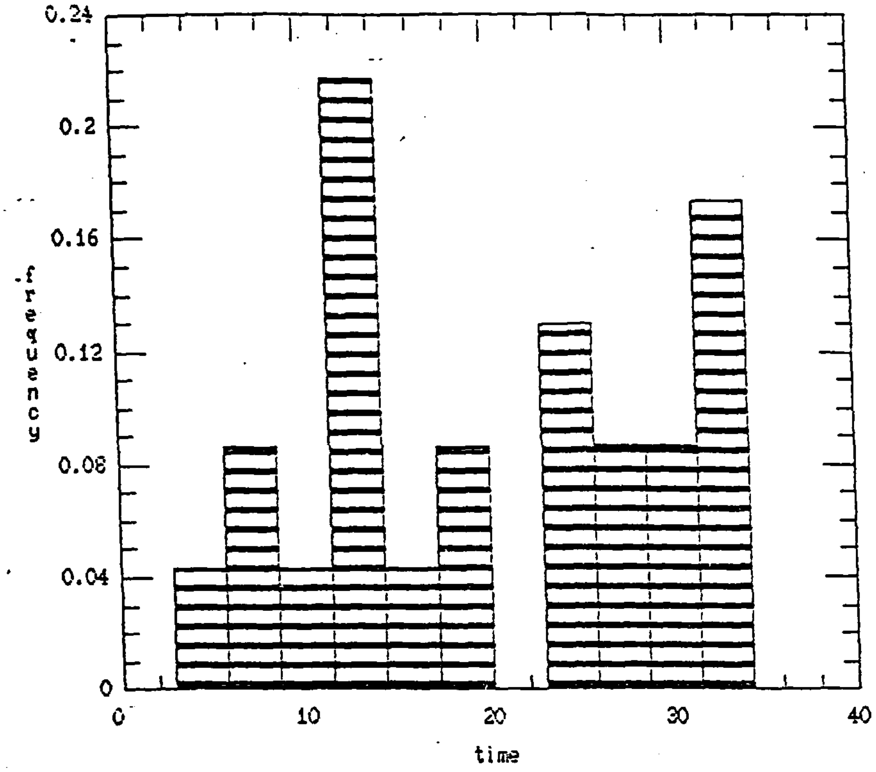


frequency
0.2
0.16
0.12
0.08
0.04
0
12
16
20
24
28
32
36
time (minutes)

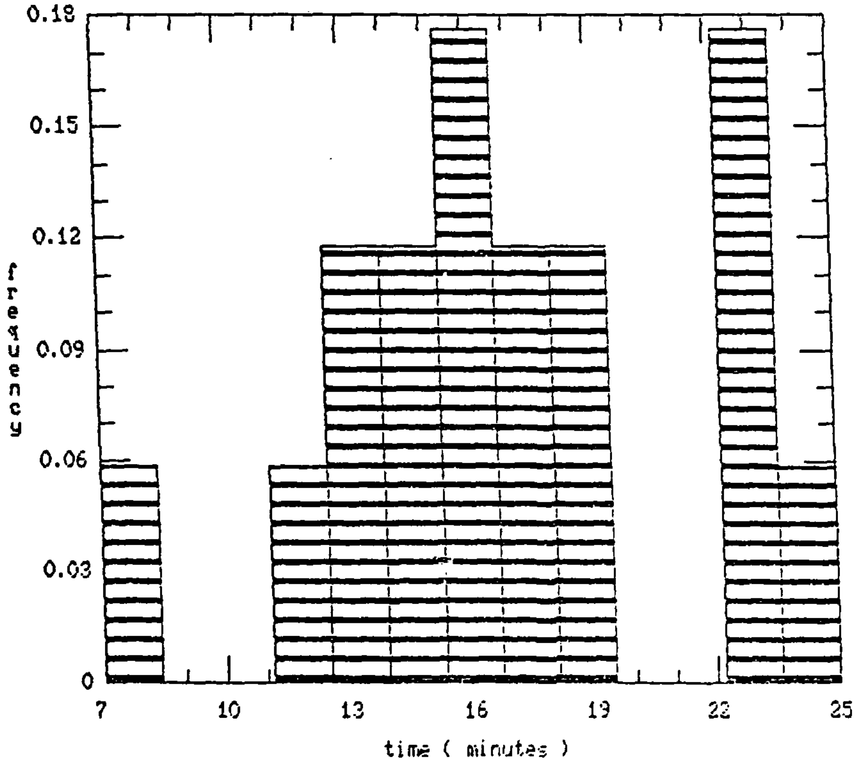
Battle 5

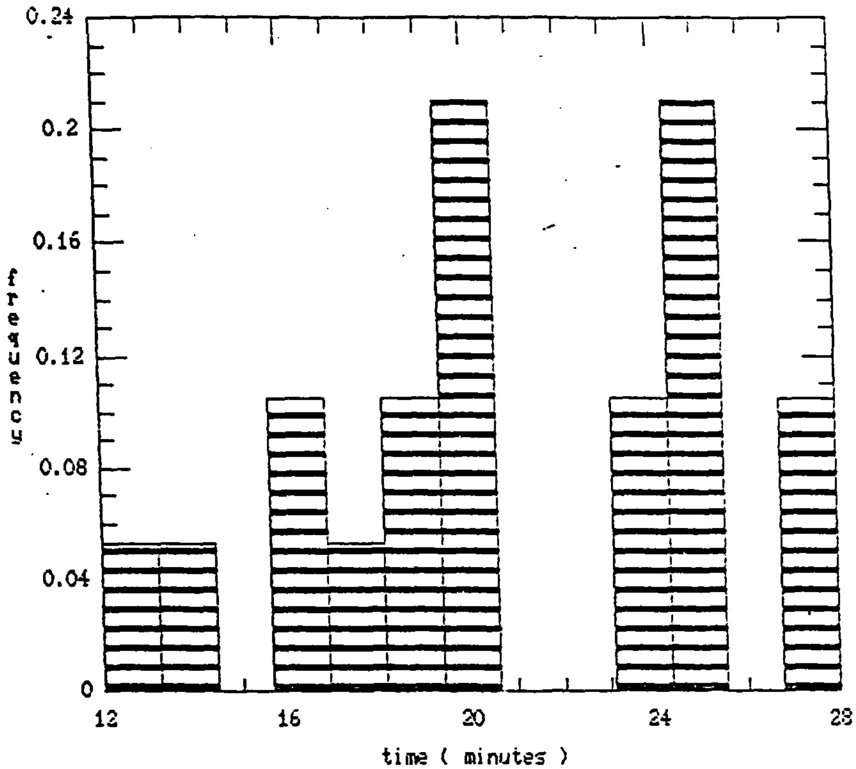


frequency
0.2
0.16
0.12
0.08
0.04
0
8
12
16
20
24
28
time (minutes)

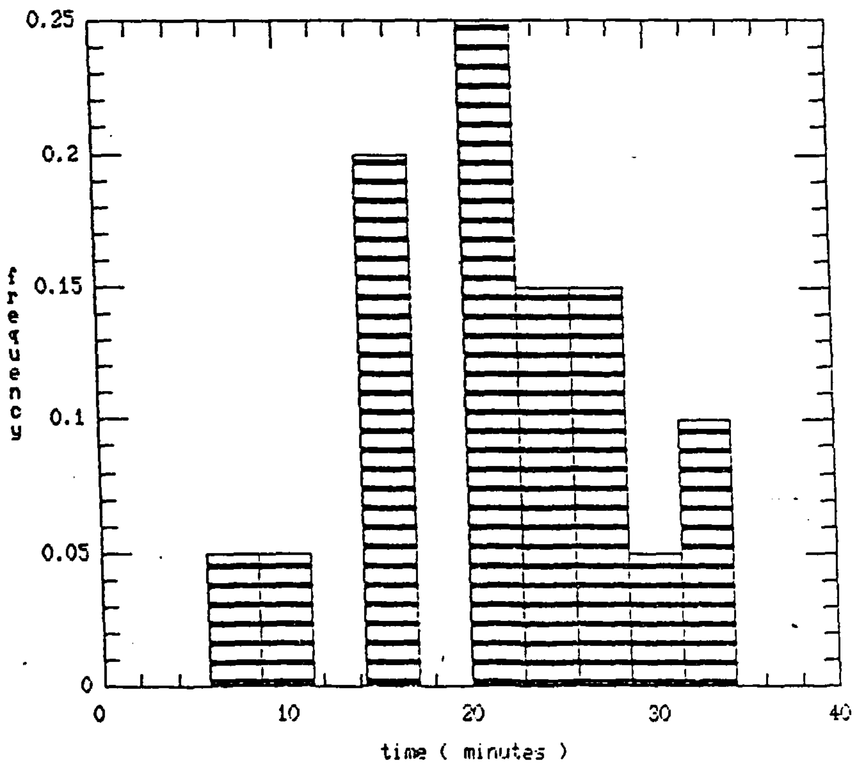

0.24
0.2
0.16
0.12
0.08
0.04
0
frequency
0
10
20
30
40
time

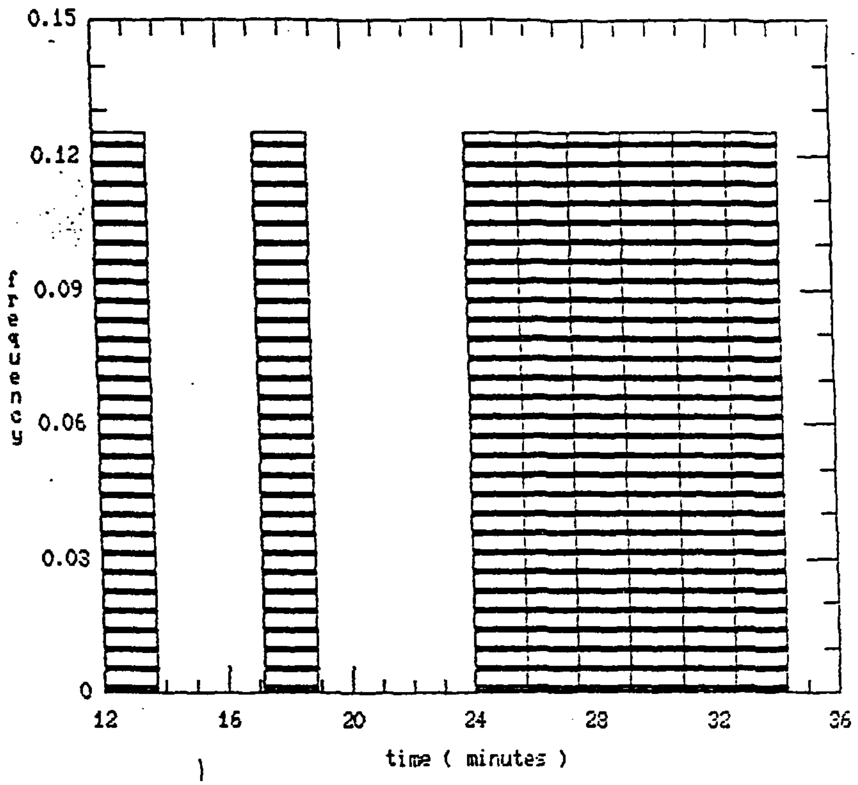
Battle 7


0.18
0.15
0.12
0.09
0.06
0.03
0
frequency
7
10
13
16
19
22
25
time (minutes)

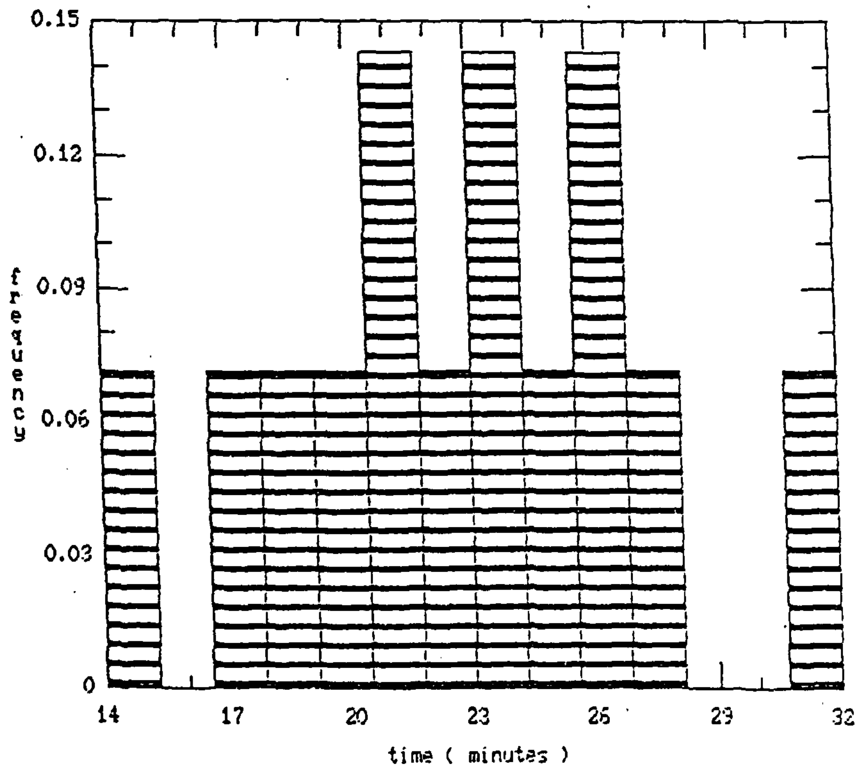

0.24
0.2
0.16
0.12
0.08
0.04
0
frequency
12
16
20
24
28
time (minutes)

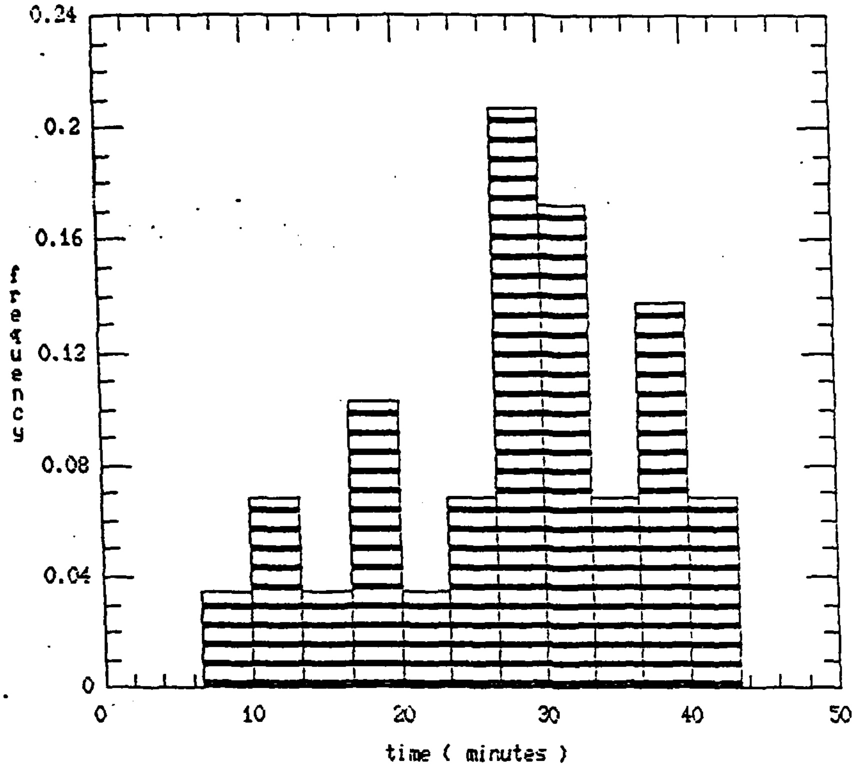
Battle 12


0.25
0.2
0.15
0.1
0.05
0
frequency
0
10
20
30
40
time (minutes)


0.15
0.12
0.09
0.06
0.03
0
frequency
12
16
20
24
28
32
36
time (minutes)

Battle 14


0.15
0.12
0.09
0.06
0.03
0
frequency
14
17
20
23
26
29
32
time (minutes)


0.24
0.2
0.16
0.12
0.08
0.04
0
frequency
0
10
20
30
40
50
time (minutes)

Battle 19

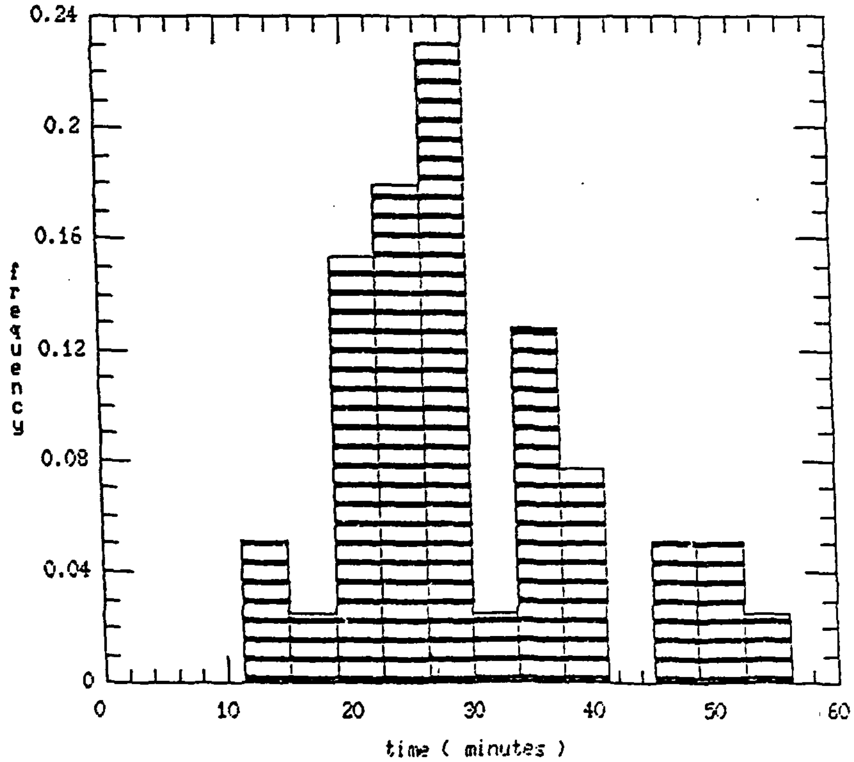

0.24
0.2
0.16
0.12
0.08
0.04
0
frequency
0
10
20
30
40
50
60
time (minutes)

FIGURE 2.

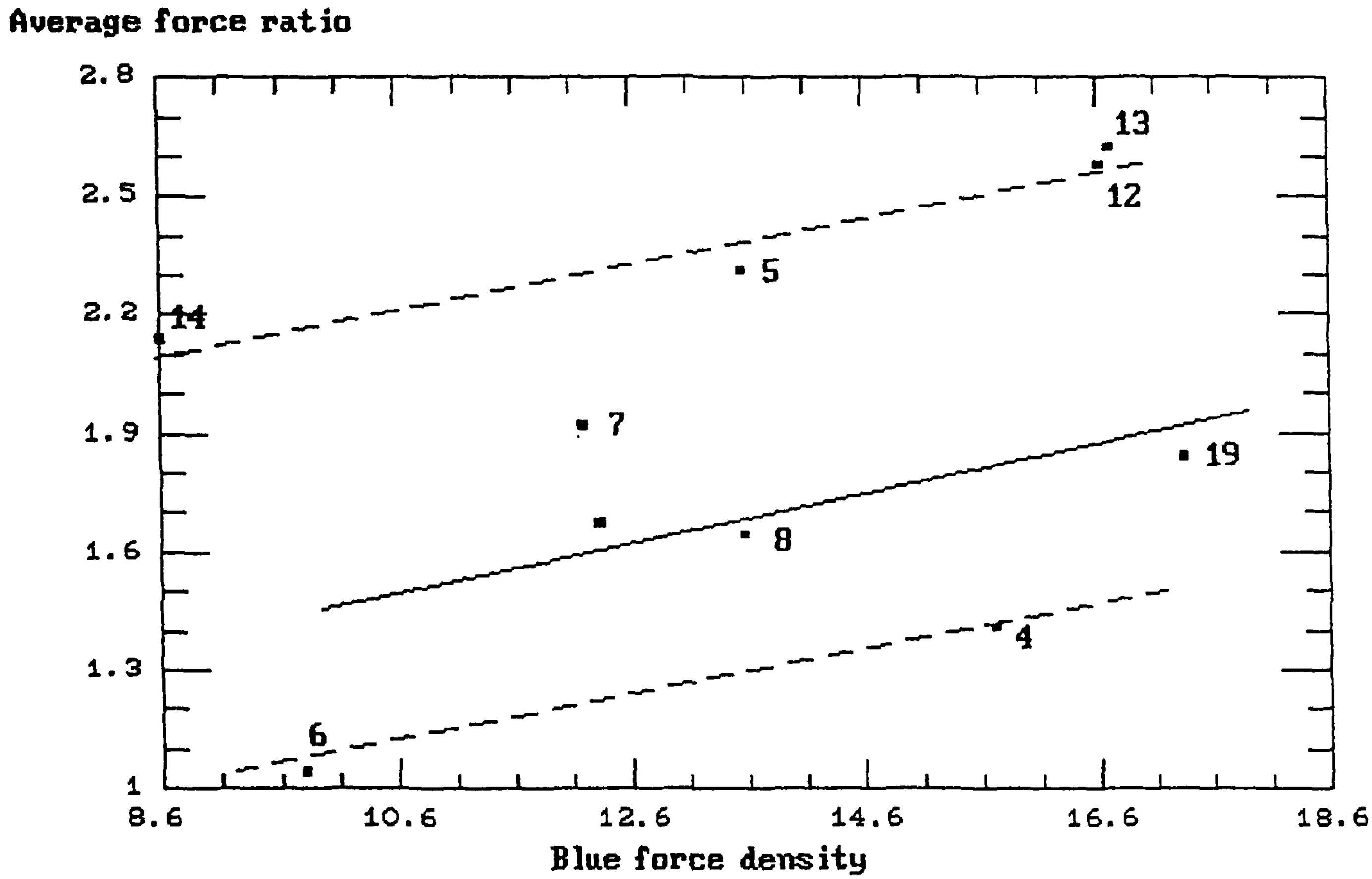
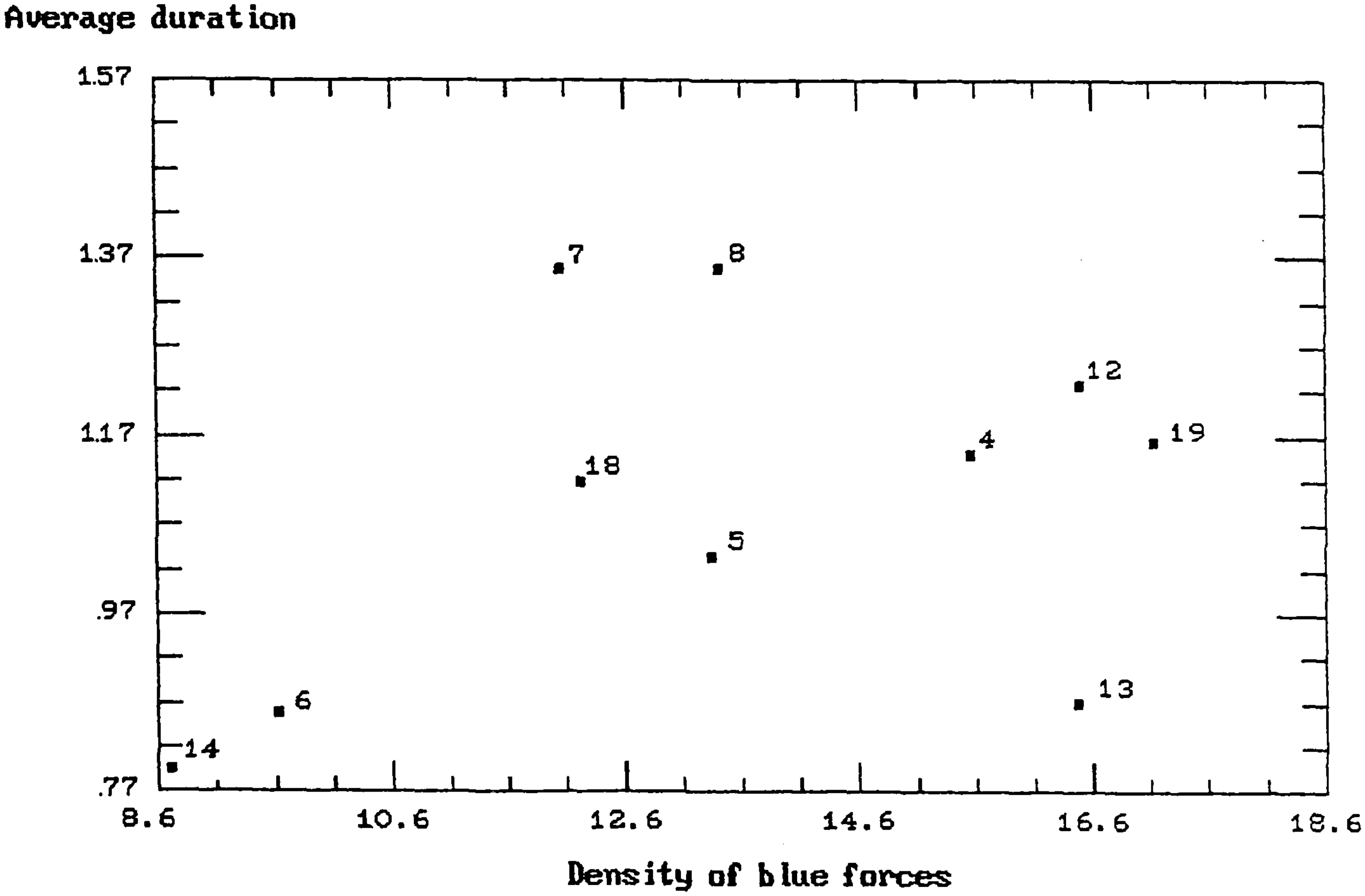

Average force ratio
2.8
2.5
2.2
1.9
1.6
1.3
1
8.6
10.6
12.6
14.6
16.6
18.6
Blue force density
13
12
5
14
7
19
8
4
6

FIGURE 3.



Average duration
1.57
1.37
1.17
.97
.77
8.6
10.6
12.6
14.6
16.6
18.6
Density of blue forces
7
8
12
19
4
18
5
6
13
14

FIGURE 4.

NETWORK GENERATION METHODOLOGIES

B1 This section discusses some network generation methodologies and issues relating to their implementation in a combat model.

B2 In previous progress reports,a number of different network generation methodologies have been discussed.However,in the light of the results of data analysis,some of these suggested methodologies have had to be reassessed.

Methods Based on the Force Ratio PDF.

B3 Initially,it was suggested that an adequate method for generating a series of minibattles for a given scenario would be simply to sample from a distribution of force ratios,specific to the scenario being analysed,over a series of time frames.This would have the advantage that 'networks' could be generated extremely quickly,and,in addition, a number of existing combat models could be adapted to use this attrition methodology.There are,however,a number of difficulties associated with this approach :

a. Data is currently only available for combat at Red Battalion - Blue Combat Team level and lower.There is no evidence to suggest that,in general,the statistical properties of a battle at this level extend to Battle Group level combat,although if each Combat Team behaves independently,and is physically removed from other members of the Battle Group,then this method may be applicable.

b. Using a statistical distribution of force ratios over time frames fails to take into account the fact that the force ratio at a given node may be highly dependent on that at previous nodes,and that,for a given battle,force ratios may exhibit positive or negative trends as a function of battle time.

c. Although characteristics of force ratio pdf's may be related to terrain and deployment properties,it is difficult to define consistent measures of these properties that give adequate results over a variety of terrain types.

On account of these difficulties,this approach may not be feasible. However,it is intended to develop a model based on this method to at least prototype level.

Statistical Methods

B4 In practical terms,the effect of a particular terrain and deployment type is that the blue and red inter-kill times (IKT) observed in a particular battle are different from those which would result from a battle where all the weapons are exposed to each other all the time.This observation suggests that it may be possible,using data from simulations,to construct a family of IKT distributions whose parameters are determined by factors related to the scenario under study.This could be achieved by defining the IKT to be a finite mixture of normal (or perhaps Gamma) distributions ie.

$$k(t) = \sum_{i=1}^{m} p_i \Phi_i(t, \alpha_i, \beta_i)$$

with parameters p_i, α_i, β_i $i=1,...,m$ determined by the specific scenario.

B5 This approach has the advantage that it would be simple to implement as a fast running simulation (and perhaps could be implemented analytically) and would not require a distinct attrition methodology to be developed.In addition,a large variety of shapes of IKT distribution could be modelled using this technique.

B6 The potential disadvantages are that data from a more highly resolved model would be required as an input and also that it would be necessary to estimate up to 3m parameters,together with m itself.In practice,although methods for estimating the parameters of a mixture have been extensively studied,methods for the estimation of the number of components in a mixture are relatively poorly developed.

B7 Furthermore,it may be necessary to estimate not k(t),but k(tb,r), the IKT conditioned on there being b,r weapons on the blue and red sides.This means that,fixing m, a total of L=3m.b.r parameters would need to be estimated,although for large values of b,r the change in k(tb,r) seems likely to be small.

B8 A number of simulation experiments are currently underway to study the statistical properties of combat models,primarily in relation to the variations in IKT as a function of inter-firing time (IFT) pdf's and force sizes.The feasibility of this approach will depend on the outcome of these studies.

A Probabilistic Approach.

B9 An alternative to the above approaches is to develop a purely probabilistic model of the combat process.

B10 Describe the combat state by the vector

$$\underline{s} = (n, \underline{f}_1, \underline{f}_2, \ldots, \underline{f}_n) \quad \text{at time } t$$

where n is the number of active minibattles at time t. $\underline{f}_1, \ldots, \underline{f}_n$ are the forces engaged at nodes 1,2, ... ,n.ie. the pair (b_i , r_i).

B11 Transitions into the state $\underline{s}$ are possible from those states with n+1 and n-1 nodes when a node is terminated or initiated, respectively.Transitions from states with n nodes occur as a result of a kill,as a result of a weapon switching to a target at the same,or at another active node,and because of detections and

LOS breaks.

B12 The transition probabilities here depend on weapon characteristics and on scenario parameters such as length and duration of LOS, attack tactics,mobile or stationary defence.

B13 This state representation can serve as the basis of a set of differential equations for the combat process,or as the basis for a simulation model.A higher level of aggregation can be achieved by allowing the basic units to be groups of red or blue weapons rather than individual weapon systems.

B14 Alternatively,represent the battle state at time t by

$$\underline{s} = (N,B,R)$$

the total number of active nodes,and the blue and red engaged force sizes respectively,and to treat the battle as consisting of N minibattles each involving either a number of weapons sampled from an appropriate probability distribution,or,at a higher level of aggregation,B/N and R/N weapons.This is clearly a much more highly aggregated approach,and is closer to the statistical procedures outlined above.

B15 As a third alternative,model the activation and termination of nodes in this way :

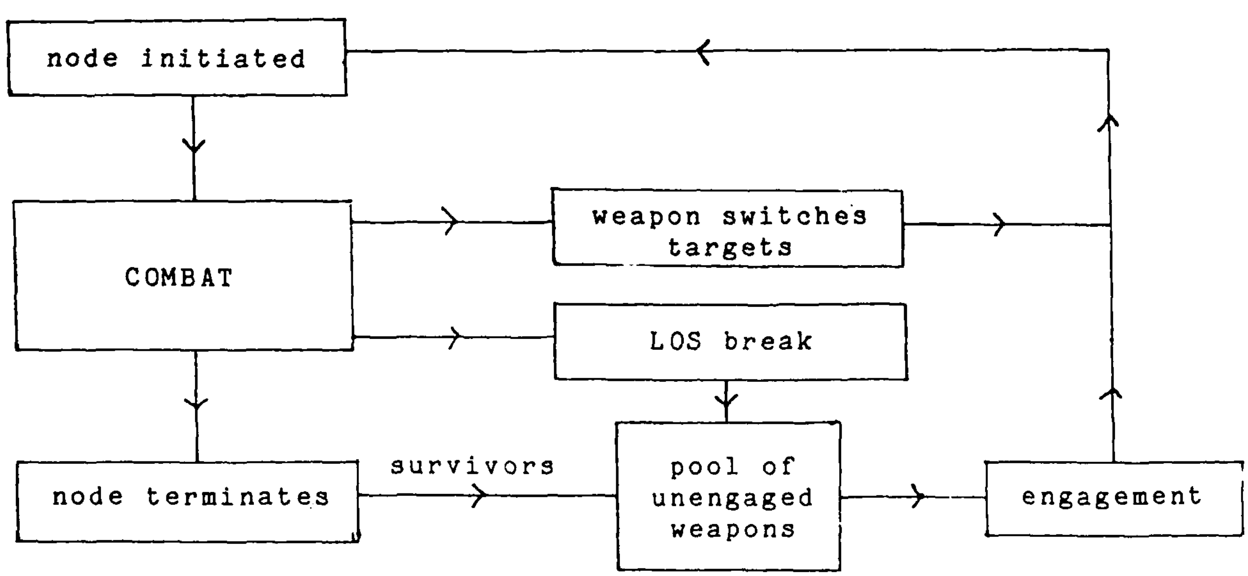
find

p(node initiated at t with b and r weapons involved)

and

p(duration of combat = x)

use simulation to set up a network,and resolve combat at a node independently of this.The relation between events is given by



B16 Any of the above will require much the same data as the computational approach,discussed below.

A Computational Approach

B17 This method takes a specific scenario and calculates the most likely decomposition arising from it.This can be achieved by computing the path of each group of weapons - a group in this context could be an individual weapon,platoon or troop - over the terrain of interest and deriving a probabilistic decomposition from this information and from weapon system parameters.This approach is,in many ways,close to that of a resolved simulation. The difference is that no resolution of combat takes place at this stage,only the occurrence of engagement opportunities is assessed. The resolution of combat takes place after the decompostion has been computed.

Data Requirements

B18 The computational and probabilistic approaches outlined above are likely to require quite detailed data relating to terrain, deployments and tactics.

B19 At this stage it is envisaged that the data requirements will be :

1. Definition of major terrain features such as hills and ridges, together with urban areas,crossings etc.

2. A deployment of defending forces at troop level together with arcs of responsibility.

3. ...inition of at least,the starting positions and orientation ... attacking forces - but preferably the route that the forces ...re to take over the defined terrain - at company level or lower.

4. The normal data relating to force composition,weapon system capabilities and basic rules of engagement.

B20 The most efficient method for defining this data depends on the intended application of the model,and on the type of computer system on which it is to be run.

B21 The intention of the team has been to produce a model which can be run on the most recent generation of IBM and compatible PC's. The most obvious approach to data input is therefore to use a graphical/mouse driven approach,with the user defining terrain primitives (hills,ridges etc.) and deploying weapons.

Conclusions

B23 All the above approaches have difficulties associated with them, mostly relating to the volume of data required to allow the model to operate.The statistical and probabilistic approaches have the advantage that no distinct attrition methodology is required to resolve combat,while the computational method is perhaps more attractive if the model is to be actively used as an assessment tool for studying force mixes,effects of terrain and tactical issues.

B24 The construction of prototype models based on the force ratio and statistical approaches discussed above will allow the relative merits these methods to be assessed.Implementation of the probabilistic and computational methodologies will take place - if necessary - after the evaluation of the prototype models is completed.

ATTRITION METHODOLOGIES

C1 A number of attrition methodologies for minibattles have been considered in the course of the work.Some aspects of the attrition problem are discussed in Annex B.,where it can be seen that certain network generation methodologies make the development of a distinct attrition methodology unnecessary.

C2 The main difficulty in attempting to assess attrition in battles involving small numbers of combatants is that the traditional Lanchester based approach is inappropriate for this type of situation,and simulations are,in general,too slow for our purposes.This means that an alternative method for the resolution of few-on-few combat is required.

C3 Initially an 'extended state space' Exponential Lanchester methodology was considered.The aim of this approach was to attempt to take explicit account of the detection processes which are usually ignored or incorporated in the kill rate in conventional Lanchester based approaches.

C4 A weapon is allowed to be in two states,waiting to detect a target,or attempting to kill it.Time in each of these states is assumed to be negative exponentially distributed (NED). Transitions between these states are then modelled in the usual way.

C5 Initial investigations suggested that the model might give good approximations to the outcomes of general (ie non-NED interfiring times) few-on-few combat.Later investigations revealed that although the model worked well for 1-on-1 and 2-on-1 situations,the results were poor for cases involving larger forces.

C6 The next approach was to attempt to model the m-on-n stochastic duel where inter firing times followed an Erlang distribution, this being in many ways a natural extension of the previous approach.

C7 The assumptions are

1. *The forces are homogeneous.*

2. There are initially B_0 and R_0 weapons on the blue and red sides.

3. For blue and red weapons

IFT	blue : $Er(\alpha,n)$	red : $Er(\beta,m)$
SSKP	p	q

4. Target selection is random.

5. Weapons switch targets instantaneously,and complete their current firing cycle if their target is killed by another weapon.

C8 The system state is then given by

$$(b_1,\ldots,b_n;r_1,\ldots,r_m) = (\underline{b},\underline{r}) \text{ ,say}$$

ie. the numbers of blue (red) weapons at stage 1,2,...,n (m) of their firing process.This then leads to a set of differential difference equations for $p_t(\underline{b},\underline{r})$,the probability that the system is in state $(\underline{b},\underline{r})$ at time t,of the form :

$$\begin{aligned}
\frac{dp_t(\underline{b},\underline{r})}{dt} = &- (n\alpha+m\beta)p_t(\underline{b},\underline{r}) + \alpha\Big(\sum_{i=1}^{n-1}(b_i+1)p_t(\underline{b}+\underline{e}_i-\underline{e}_{i+1},\underline{r})\,\delta(b_{i+1},0)\Big) \\
&+ \beta\Big(\sum_{i=1}^{m-1}(r_i+1)p_t(\underline{b},\underline{r}+\underline{e}_i-\underline{e}_{i+1})\,\delta(r_{i+1},0)\Big) \\
&+ (1-q)\beta(r_n+1)p_t(\underline{b},\underline{r}+\underline{e}m-\underline{e}_1)\delta(r_1,0) \\
&+ (1-p)\alpha(b_n+1)p_t(\underline{b}+\underline{e}n-\underline{e}_1,\underline{r})\delta(b_1,0) \\
&+ \alpha p\delta(b_1,0)(b_n+1)\Big(\sum_{i=1}^{m-1}p_t(\underline{b}+\underline{e}_n-\underline{e}_1,\underline{r}+\underline{e}_i)(r_i/R)\Big) \\
&+ \beta q\delta(r_1,0)(r_m+1)\Big(\sum_{i=1}^{n-1}p_t(\underline{b}+\underline{e}_i,\underline{r}+\underline{e}_m-\underline{e}_1)(b_i/B)\Big)
\end{aligned}$$

where $R = \sum r_i$, $B = \sum b_i$, $\delta(x,0) = 0$ if $x=0$ and 1 otherwise

and $\underline{e}_i = (0,0,\ldots,1,\ldots,0)$ ie. the zero vector,with a 1 in the ith position

C9 The probability of having a total of B and R survivors at time t,$P_t(B,R)$ is then given by

$$\sum\sum p_t(\underline{b},\underline{r})$$

where the summation is over all values of $\underline{b}$ and $\underline{r}$ that sum to B,R.

C10 The above equation can be reduced to a second order PDE in m+n+1 dimensions,for the probability generating function of the process.

C11 The complexity of the above system of equations and the resulting PDE suggests that efficient analytical or numerical solutions are not a real possibility.However studies relating to this problem are likely to continue independently of the current research,since the approach can be adapted to take account ofdetection processes, inhomogeneous forces,etc.

C12 The final approach so far considered is the use of approximate methods.

C13 A simulation has been constructed to provide data for comparison of approximation methods and this simulation may also be used to determine those factors which have a significant effect on the outcome of a battle.

C14 Gafarian has studied approximate methods involving the use of an Inhomogeneous Poisson Process approximation to the inter-kill time (IKT),and has obtained good results,although these appear not to have been published at the time of writing.

C15 Other methods being studied are those which approximate the conditional IKT distribution by series expansion methods or by data fitting.Such an approach would allow a fast running simulation or an analytical model to be constructed,and relate to the IKT network generation method discussed in Annex B.

C16 In conclusion,it appears that the most effective means of resolving attrition in a minibattle will use some form of approximation to the inter-kill time pdf,although it is not possible to say at this stage which of the available methods will be the most appropriate.In addition,the continuing work of Ancker and Gafarian on general stochastic duel models will significantly influence any conclusions which may be reached in this section of the research.